BITS Pilani
K K Birla Goa Campus

MECHANICS OF SOLIDS
ME F211

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Chapter-5

Stress-Strain-Temperature Relations

Stress-Strain-Temperature Relations

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Stress-Strain-Temperature Relations

Introduction

Elastic deformation:

It is the deformation which exist when load applied and it disappears as soon as the load is released.

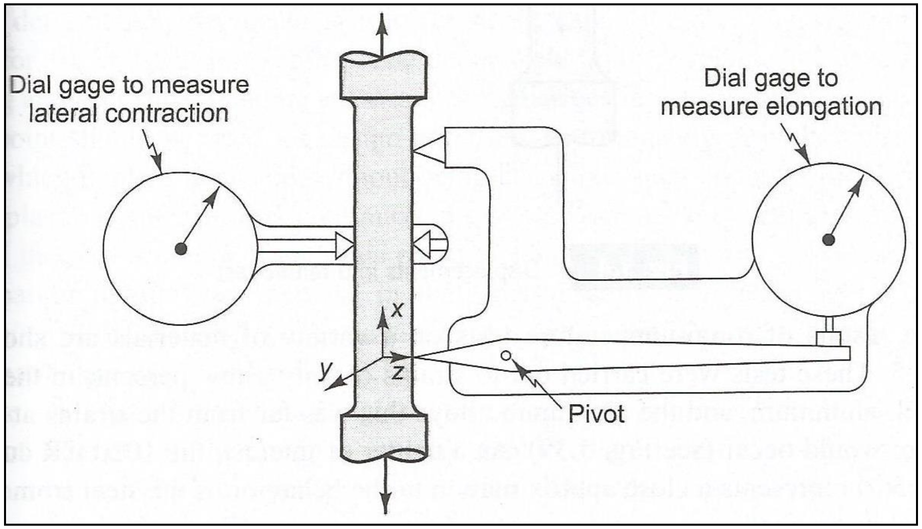
Plastic Deformation:

when load is applied beyond elastic range, and the deformation does not disappears after the load is released.

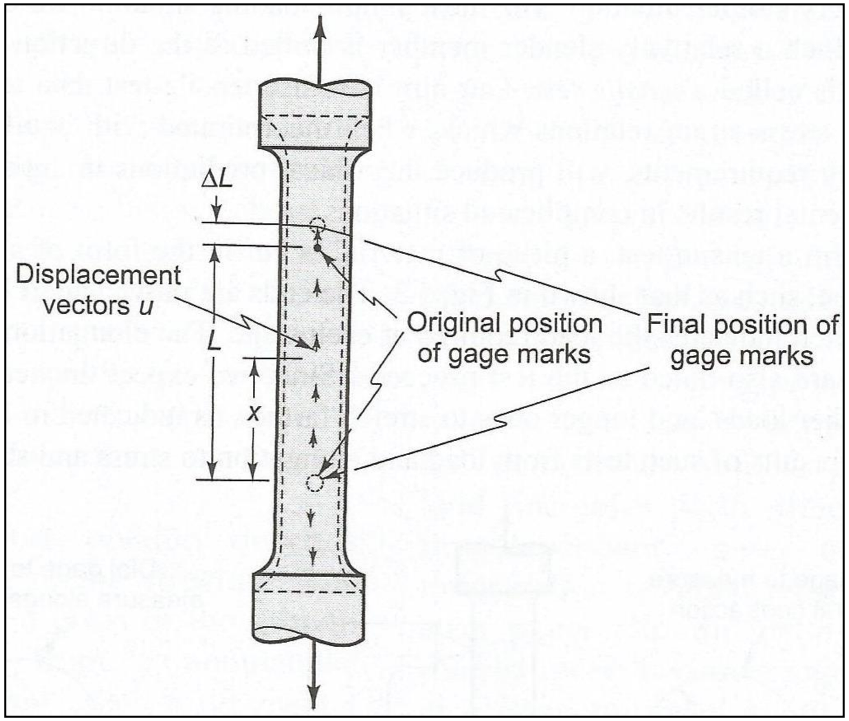
A Ductile materials is one which the plastic deformation is much larger than elastic deformation

Stress-Strain-Temperature Relations

Tensile Test



Tensile Test

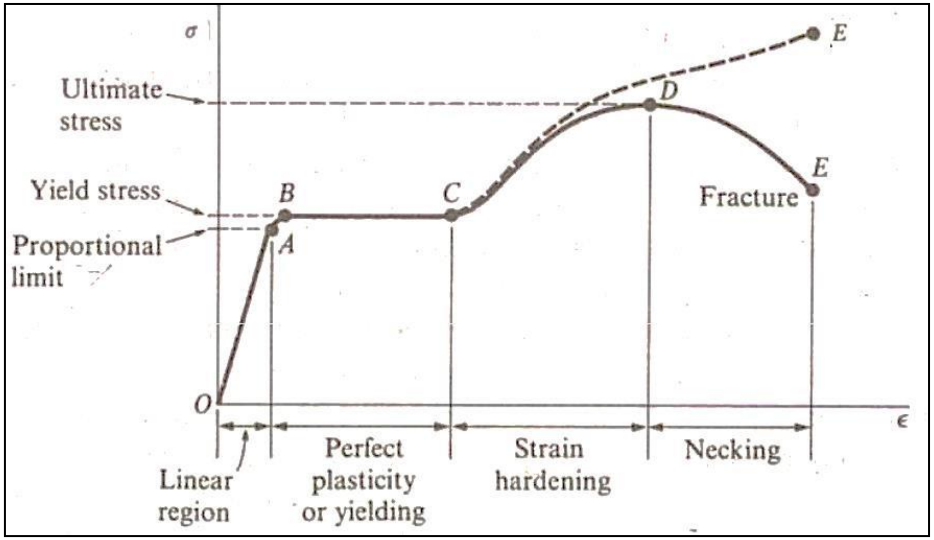


Displacements in Tensile Test

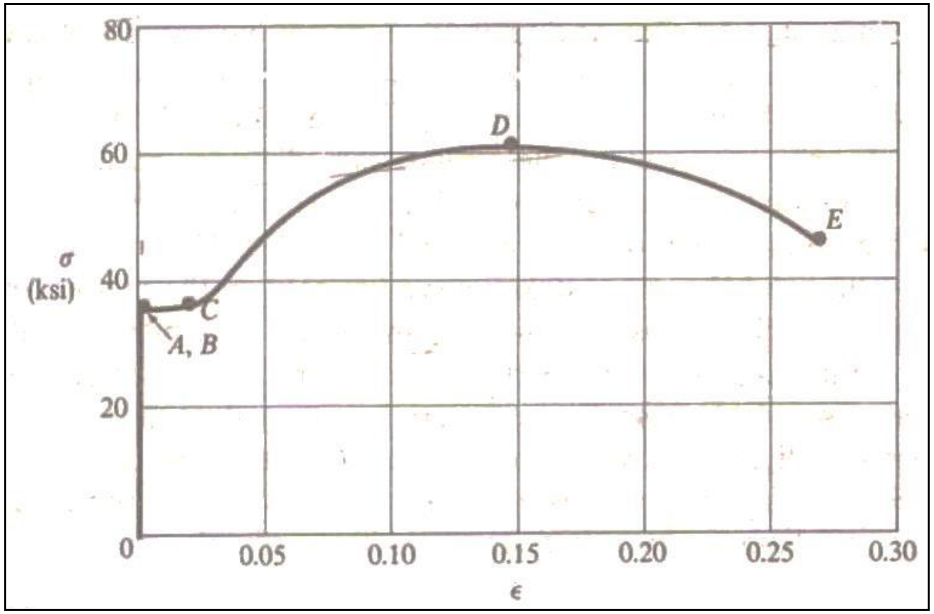
Stress-Strain-Temperature Relations

Stress- Strain diagram

Stress-strain diagram for a structural steel in tension



Not to scale

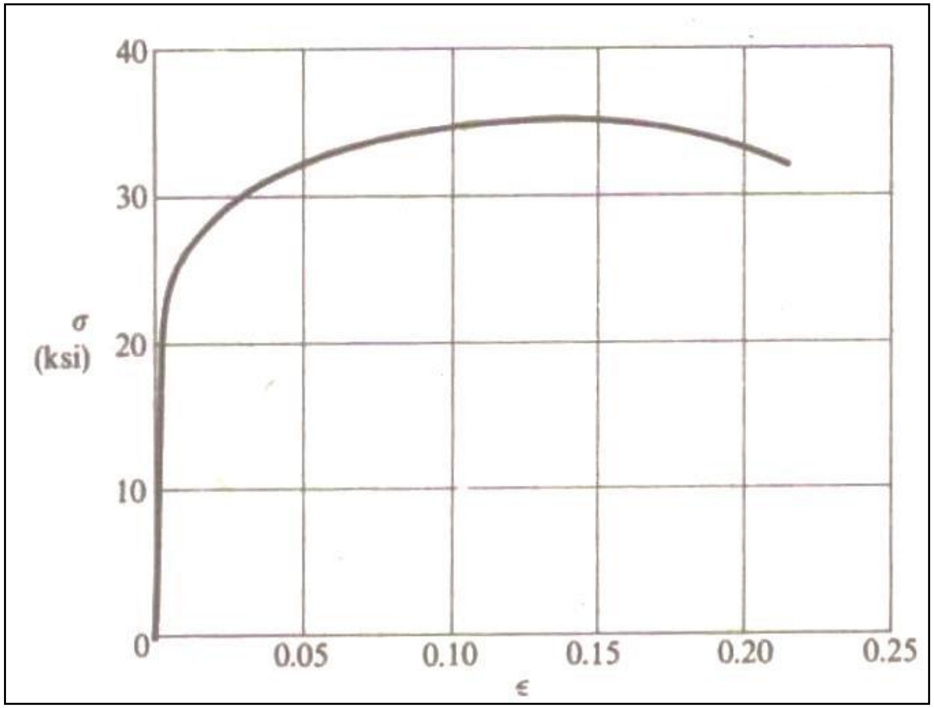


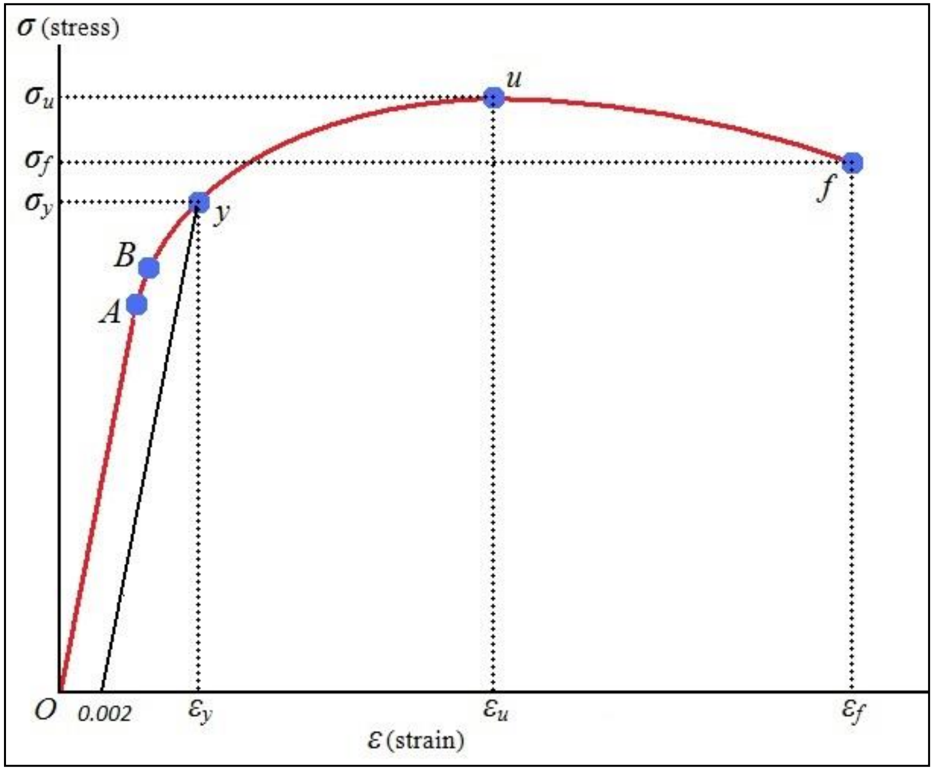
Draw to scale

Stress-Strain-Temperature Relations

Stress- Strain diagram

Typical stress- strain diagram for an aluminum alloy



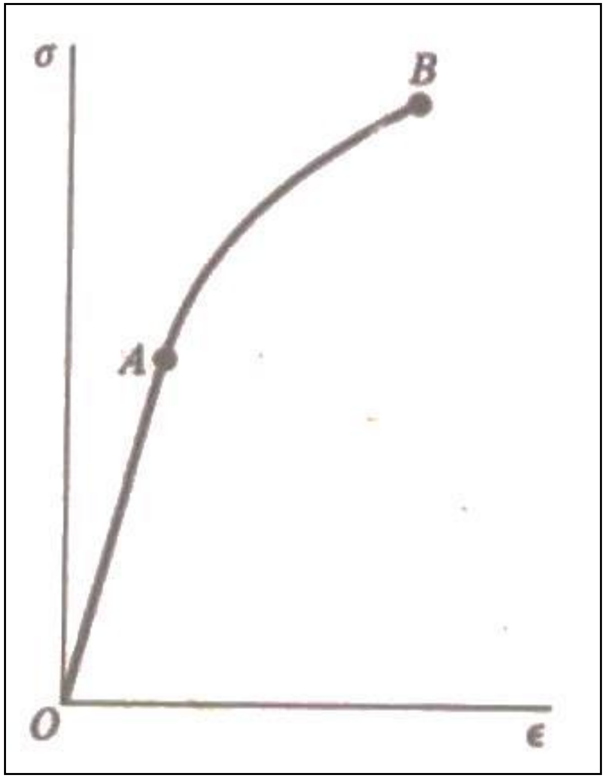


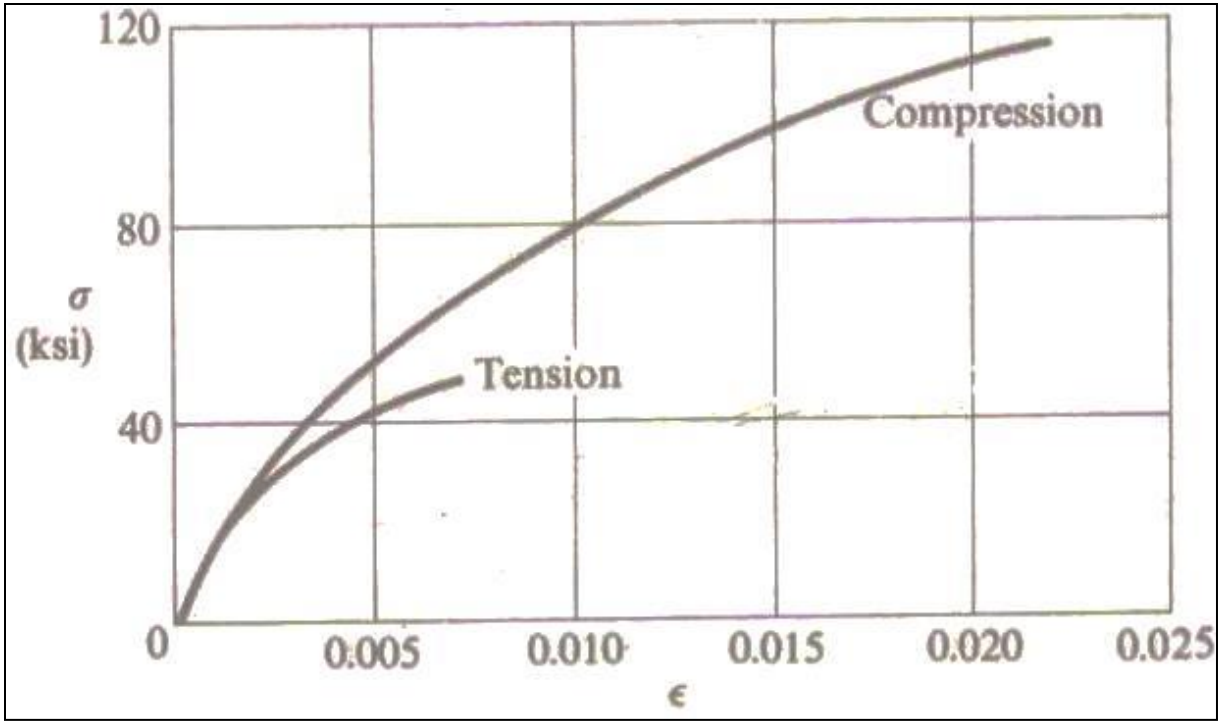
Offset yield point (proof stress)

Stress-Strain-Temperature Relations

Stress- Strain diagram

Typical stress-strain diagram for brittle material



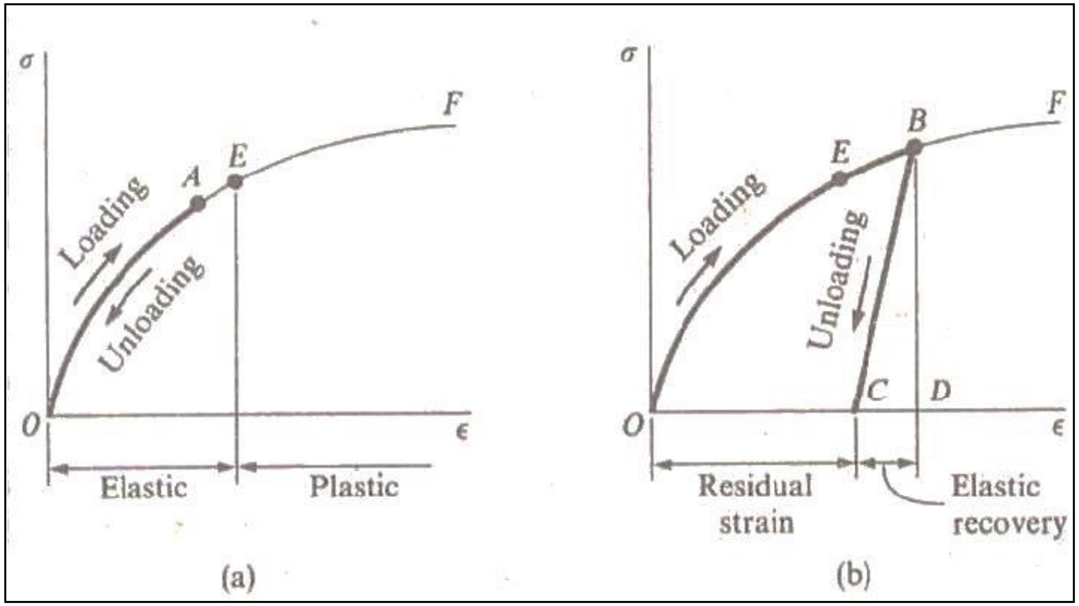


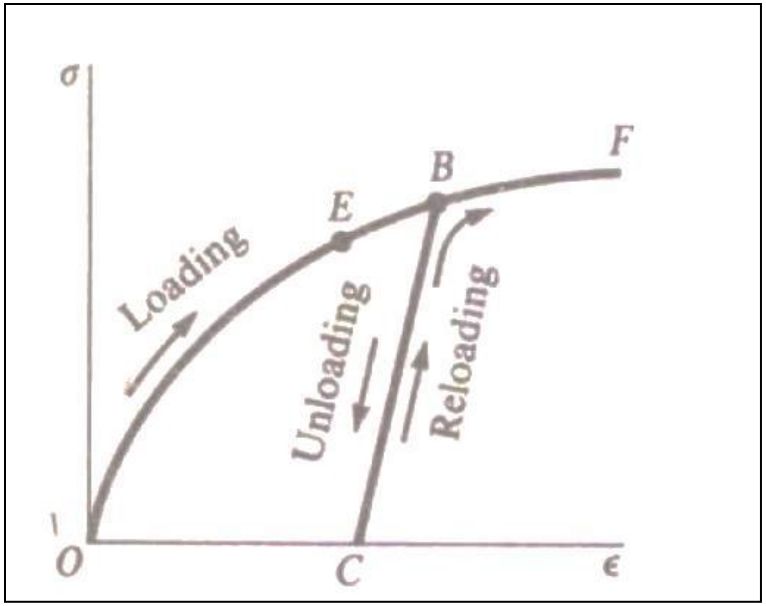
Stress-strain diagram for a cast iron in tension and compression

Stress-Strain-Temperature Relations

Stress- Strain diagram

Loading, Unoading and Reloading



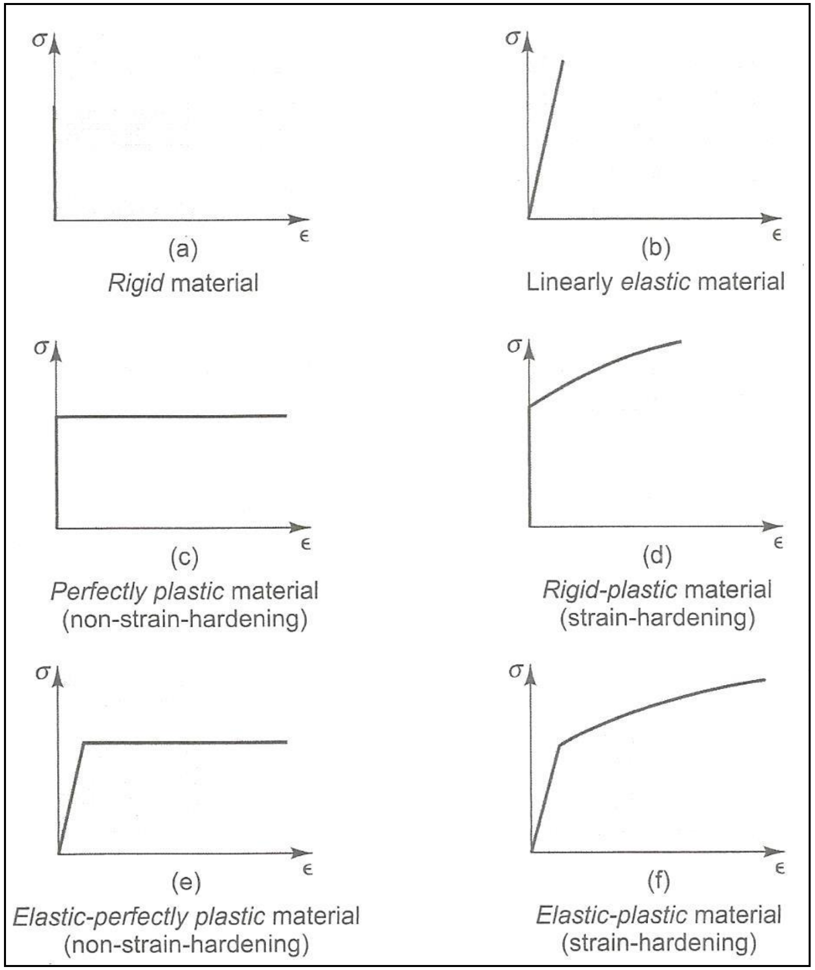


(a) Elastic behaviour
(b) Partially elastic behaviour

Reloading of a material and raising of the yield stress

Stress-Strain-Temperature Relations

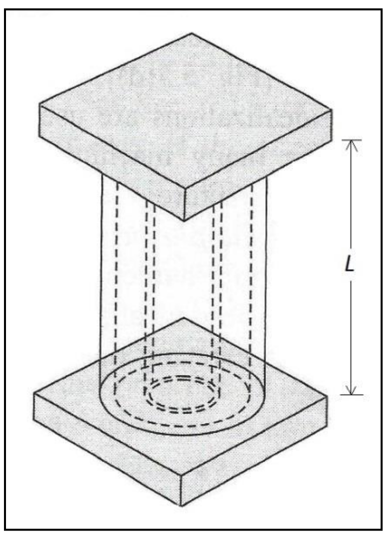
Idealizations of Stress-Strain Curves

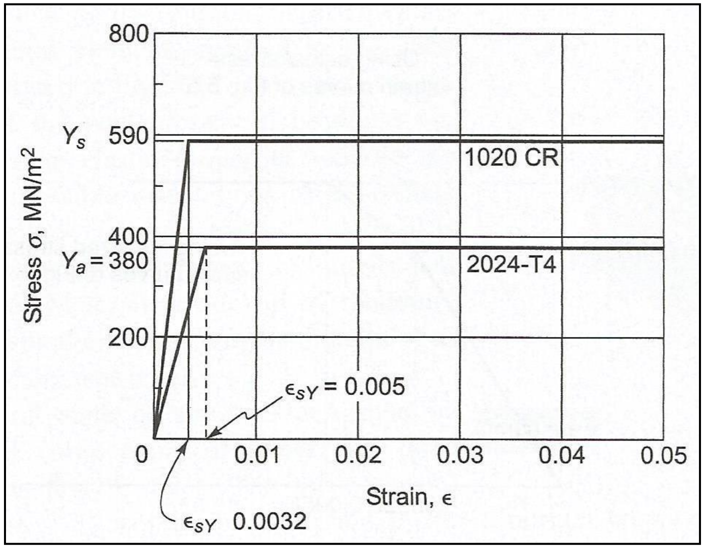


Stress-Strain-Temperature Relations

Example:

Two Coaxial tubes, the inner one of 1020 CR steel and cross sectional area A_s, and the outer one of 2024-T4 aluminum alloy and of area A_a are compressed between heavy , flat end plates as shown in figure. We wish to determine the load-deflection curve of the assembly as it is compressed into the plastic region by an axial force *P*. The idealized stress strain curves are shown for both the tube materials in the second figure





Stress-Strain-Temperature Relations

Solution

In the range $0 \le \varepsilon \le 0.0032$,

$$\sigma_s = E_s\varepsilon_s = E_s\varepsilon \quad \text{and} \quad \sigma_a = E_a\varepsilon_a = E_a\varepsilon$$

where $$E_s = \frac{590}{0.0032} = 184GPa$$

$$E_a = \frac{380}{0.005} = 76GPa$$

In the range $0.0032 \le \varepsilon \le 0.005$,

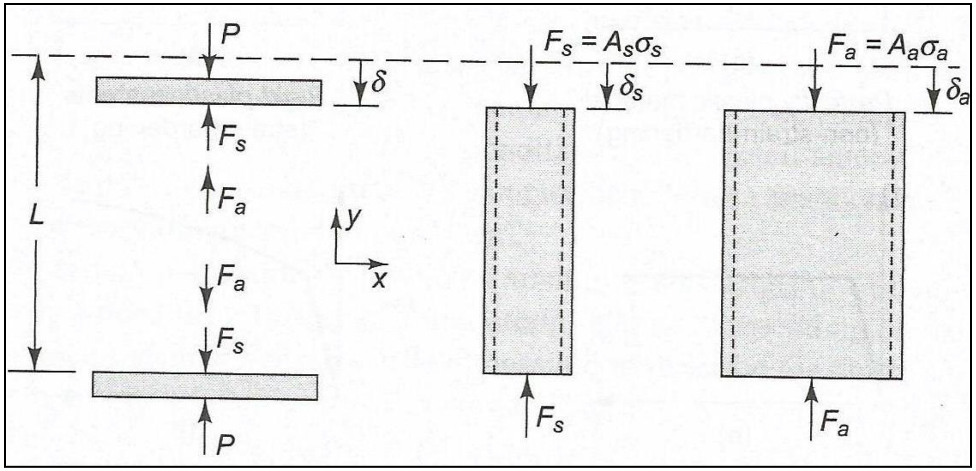
$$\sigma_s = Y_s = 590MPa \quad \text{and} \quad \sigma_a = E_a\varepsilon_a = E_a\varepsilon$$

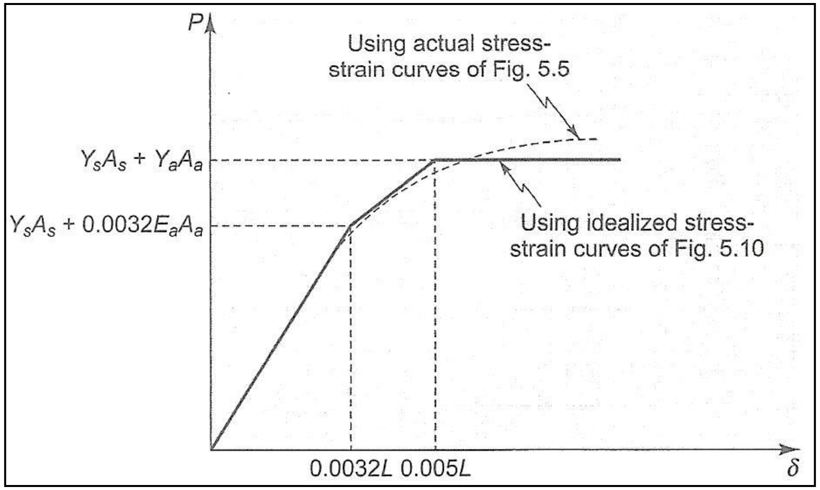
In the range $0.005 \le \varepsilon$,

$$\sigma_s = Y_s = 590MPa \quad \text{and} \quad \sigma_a = Y_a = 380MPa$$

Equilibrium

$$\sum F_y = \sigma_s A_s + \sigma_s A_a - P = 0$$

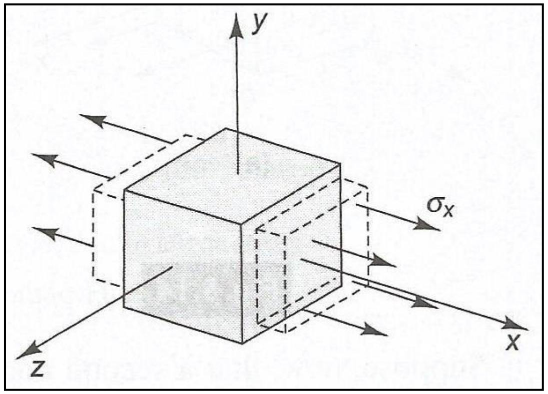




Stress-Strain-Temperature Relations

Elastic Stress- Strain Relations

- Assuming that the material is isotropic and homogeneous.
- Consider an element on which there is only one component of normal stress acting, as shown in figure.
- For linear elastic solid, stress is directly proportional to strain.
- Uniaxial tension (compression) test shows that lateral compressive (extensional) strain is a fixed fraction of longitudinal extensional (compressive) strain.
- This fixed fraction is known as *Poisson's ratio* and given by symbol ν.



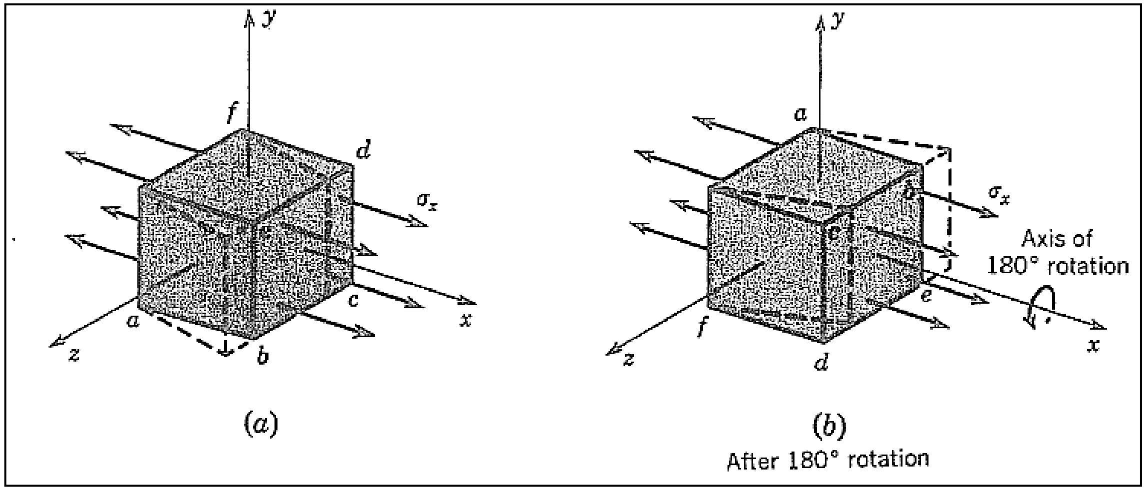
$$\varepsilon_x = \frac{\sigma_x}{E}$$

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -\nu\frac{\sigma_x}{E}$$

Stress-Strain-Temperature Relations

Elastic Stress- Strain Relations

- For isotropic material normal stress does not produce shear strain.



Similarly only normal stress σ_y acts

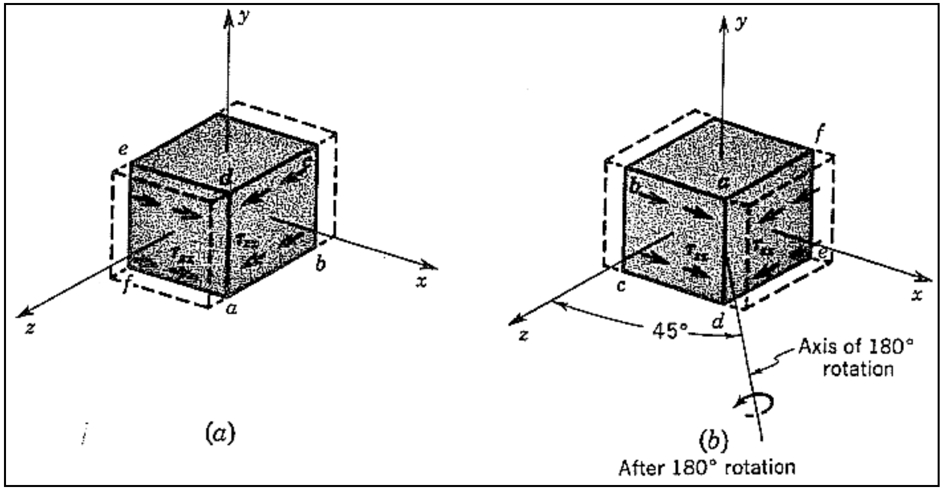
$$\varepsilon_y = \frac{\sigma_y}{E} \quad \text{and} \quad \varepsilon_x = \varepsilon_z = -\nu\varepsilon_y = -\nu\frac{\sigma_y}{E}$$

Analogous results are obtained for the strain due to σ_z.

Stress-Strain-Temperature Relations

Elastic Stress- Strain Relations

- For isotropic material shear stress does not produce normal strain.
- Each shear-stress component produces only its corresponding shear-strain component.
- For isotropic materials, shear stress is directly proportional to the shear strain. The proportionality constant is *G* (shear modulus).



$$\gamma_{xy} = \frac{\tau_{xy}}{G} \qquad \gamma_{yz} = \frac{\tau_{yz}}{G} \qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

Stress-Strain-Temperature Relations

Elastic Stress- Strain Relations

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

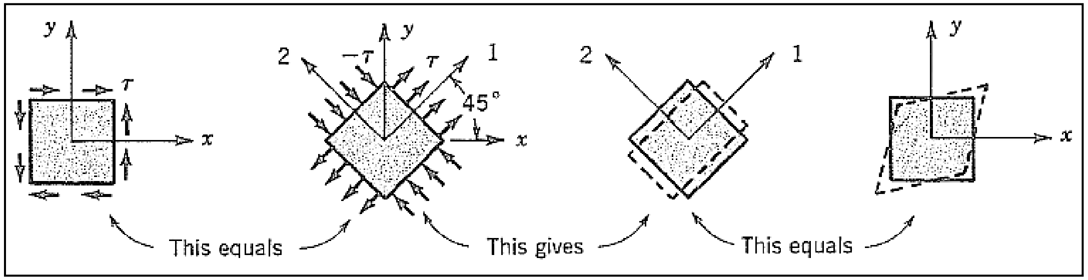
$$\gamma_{xy} = \frac{\tau_{xy}}{G} \qquad \gamma_{yz} = \frac{\tau_{yz}}{G} \qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

Relation between *E, G* and ν

$$\varepsilon_I = \frac{\tau(1+\nu)}{E} \text{ and } \varepsilon_{II} = -\frac{\tau(1+\nu)}{E}$$

$$\gamma_{xy} = \frac{\tau}{G} = \varepsilon_I - \varepsilon_{II} = \frac{2\tau(1+\nu)}{E}$$

$$E = 2G(1+\nu)$$



Stress-Strain-Temperature Relations

Thermal Strain

- The effect of temperature on elastic constants for many materials is small for temperature change of a hundred degree centigrade and we will not consider for our study
- The strain due to temperature change in the absence of stress is called thermal strain and it is denoted by superscript *t* on strain symbol (ε^t)
- For isotropic material, thermal strain is pure expansion or contraction with no shear components.
- Thermal strain due to a change in temperature from T_o to T is

$$\varepsilon_x{}^t = \varepsilon_y{}^t = \varepsilon_z{}^t = \alpha\left(T - T_o\right)$$
$$\gamma_{xy}{}^t = \gamma_{yz}{}^t = \gamma_{xz}{}^t = 0$$

- The quantity α is coefficient of linear expansion.

Stress-Strain-Temperature Relations

Thermal Strain

- Denoting the elastic strain due to stress by ε^e and the thermal part by ε^t, the total strain is

$$\varepsilon = \varepsilon^e + \varepsilon^t$$

- If material is rigidly restrained so that no strain is possible, the elastic part of the strain will be the negative of the thermal strain.

Stress-Strain-Temperature Relations

Complete Equations of Elasticity

- Equilibrium Equations
- Geometric Compatibility
- Stress-strain Temperature Relations

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\left(T - T_o\right)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\left(T - T_o\right)$$

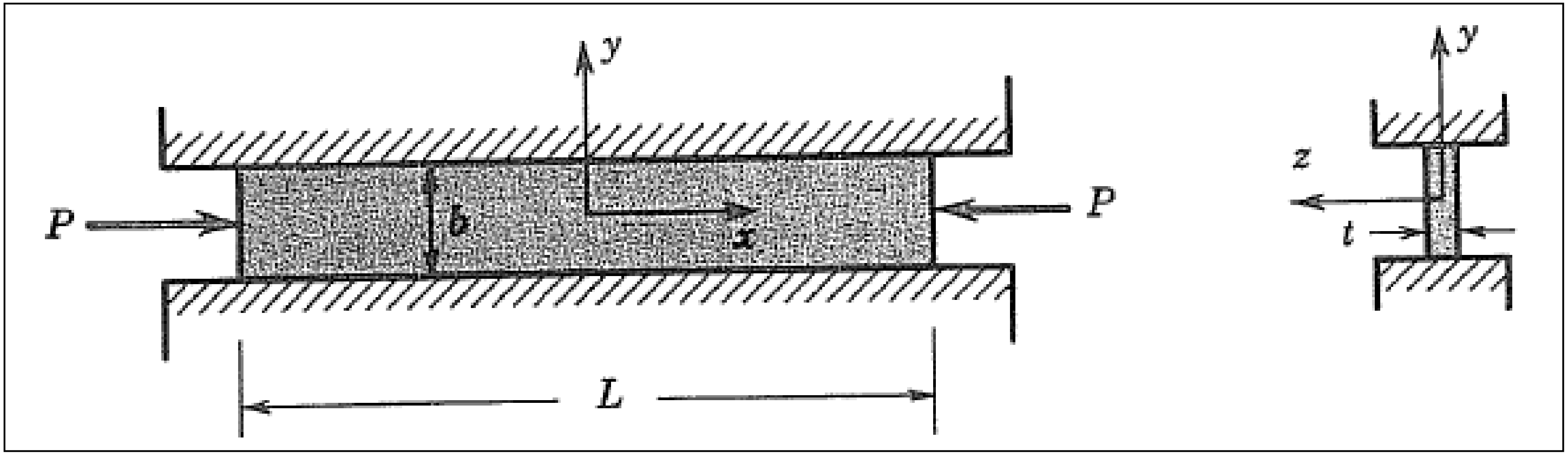
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\left(T - T_o\right)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \qquad \gamma_{yz} = \frac{\tau_{yz}}{G} \qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

Stress-Strain-Temperature Relations

Example

A long, thin plate of width *b*, thickness *t*, and length *L* is placed between two rigid walls a distance *b* apart and is acted on by an axial force *P*, as shown in figure. We wish to find the deflection of the plate parallel to the force *P*.



Stress-Strain-Temperature Relations

Solution

Assumptions

1. The axial force P results in an axial normal stress uniformly distributed over the plate area, including the end areas.
2. There is no normal stress in the thin direction (plane stress in x-y plane).
3. There is no deformation in the y direction, that is, $\varepsilon_y = 0$ (plane strain in x-z plane).
4. There is no friction force at the walls.
5. The normal stress of contact between the plate and wall is uniform over the length and width of the plate.

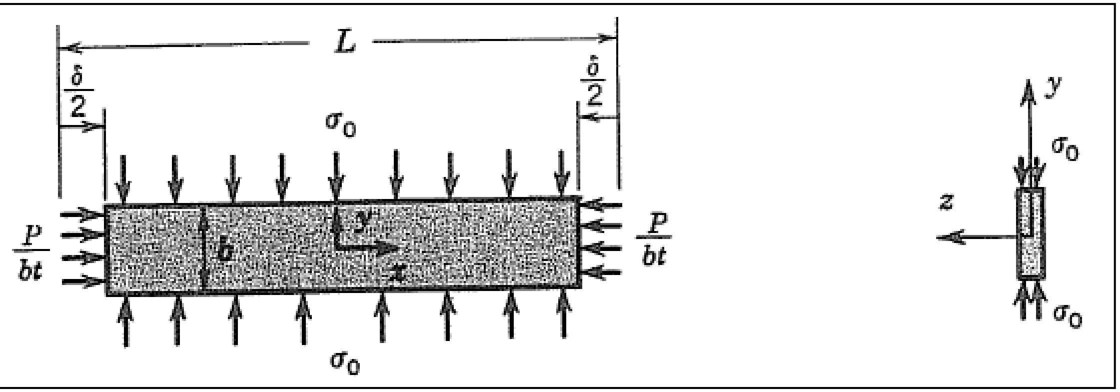
Stress-Strain-Temperature Relations

Solution

Equilibrium

$$\sigma_x = -\frac{P}{bt} \qquad \sigma_y = -\sigma_o \qquad \sigma_z = 0$$

$$\tau_{xy} = \tau_{yz} = \tau_{xz} = 0$$



Geometric Compatibility

$$\varepsilon_y = 0 \text{ and } \varepsilon_x = -\frac{\delta}{L}$$

Stress Strain Relations

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right) \qquad \varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right) \qquad \varepsilon_z = -\frac{\nu}{E}\left(\sigma_y + \sigma_x\right)$$

$$\gamma_{xy} = \gamma_{yz} = \gamma_{xz} = 0$$

After solving

$$\sigma_y = \nu\sigma_x = -\nu\frac{P}{bt} \qquad \delta = \left(1-\nu^2\right)\frac{PL}{Ebt} \qquad \varepsilon_z = \nu\left(1+\nu\right)\frac{P}{Ebt} = \frac{\nu}{1-\nu}\frac{\delta}{L}$$

Stress-Strain-Temperature Relations

Problem

The stress in a flat steel plate in a condition of plane stress are, σ_x = 130MPa; σ_y = -70MPa and τ_{xy} = 80MPa.

Find the magnitude and orientation of the principal strains in the plane of the plate. Find also the magnitudes of the third principal strain

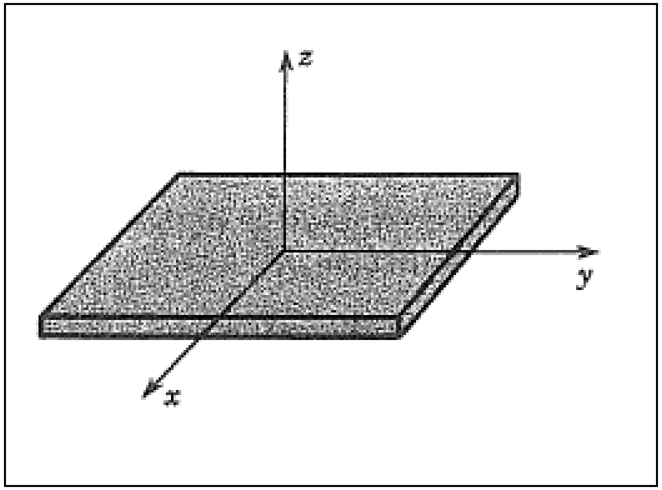
Stress-Strain-Temperature Relations

Problem

In a flat steel plate which is loaded in the xy plane, it is known that

σ_x = 145MPa; τ_{xy} = 42MPa and $\varepsilon_z = -3.6\times10^{-4}$.

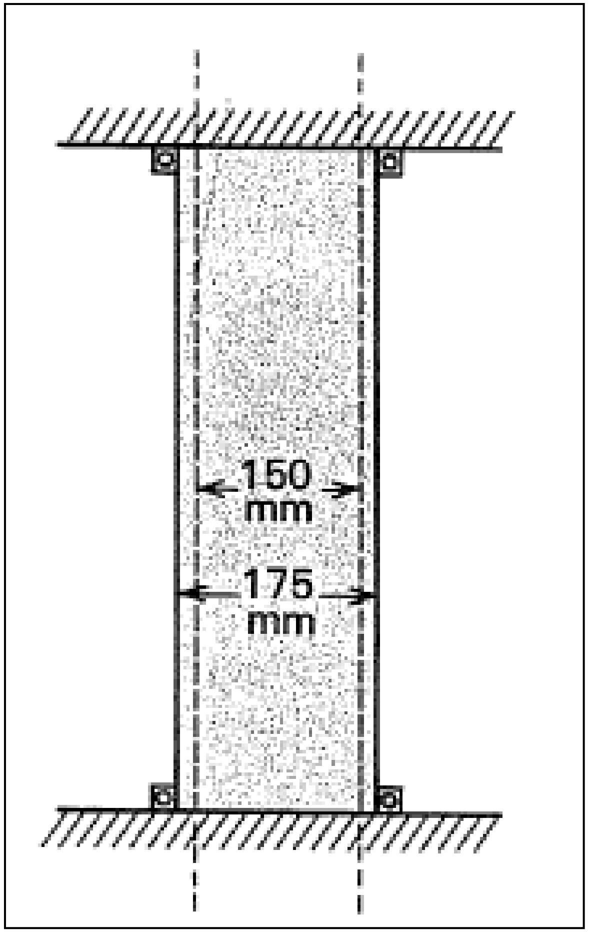
What is the value of the stress σ_y?



Stress-Strain-Temperature Relations

Problem

A steel pipe is held by two fixed supports as shown in figure. When mounted, the temperature of the pipe was 20ºC. In use, however, cold fluid moves through the pipe, causing it to cool considerably. If we assume that the pipe has uniform temperature of -15ºC and if we take the coefficient of linear expansion to be 12×10^{-6}/ºC for this temperature range, determine the state of stress and strain in the central portion of the pipe as a result of this cooling. Neglect the local end effects near the supports and neglect body forces and fluid pressure and drag forces.



Stress-Strain-Temperature Relations

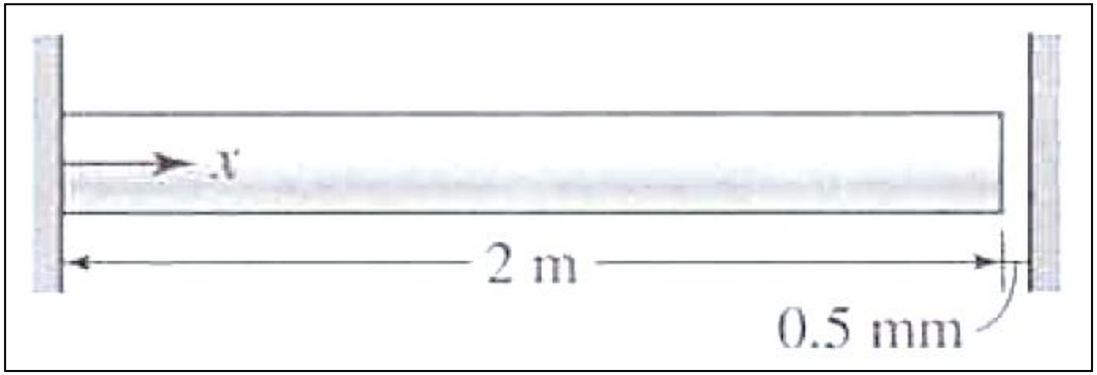
Problem

It is desired to produce a tight fit of a steel shaft in a steel pulley. The internal diameter of the hole in the pulley is 24.950mm, while the outside diameter of the shaft is 25.000mm. The pulley will be assembled on the shaft by either heating the pulley or cooling the shaft and then putting the shaft in the pulley hole and allowing the assembly to reach uniform temperature. Is it more effective to heat the pulley or to cool the shaft? What temperature change would be required in each case to produce a clearance of 0.0025 mm for easy assembly?

Stress-Strain-Temperature Relations

Problem

A circular bar (E = 200GPa, ν = 0.32, and $\alpha = 11.7\times10^{-6}$ /°C) has a diameter of 100mm. The bar is built into a rigid wall on the left, and a gap of 0.5 mm exists between the right wall and the bar prior to an increase in temperature as shown in figure. Temperature of the bar is increased uniformly by 80°C. Determine the average axial stress and the change in the diameter of the bar.



Stress-Strain-Temperature Relations

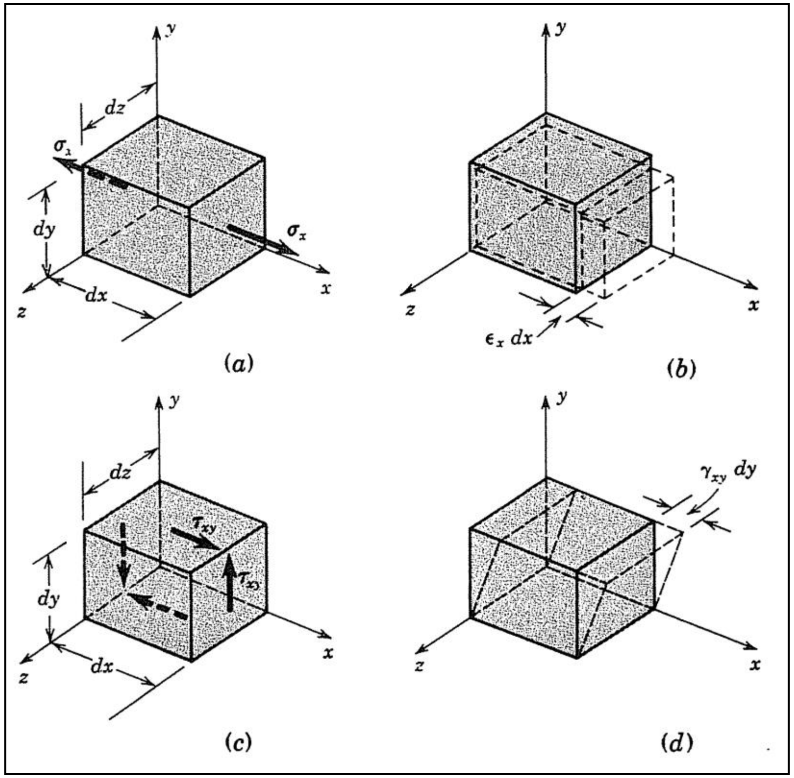
Strain Energy in an Elastic Body

The energy stored in a body due to deformation is called strain energy.

For small element shown in figure (*a*),

$$dU = \frac{1}{2}(\sigma_x dydz)(\varepsilon_x dx) = \frac{1}{2}\sigma_x \varepsilon_x dV$$

$$U = \frac{1}{2}\int_V \sigma_x \varepsilon_x dV = \frac{1}{2}\left(\frac{P}{A}\right)\left(\frac{\delta}{L}\right)\int_V dV = \frac{1}{2}P\delta$$



Similarly considered small element shown in figure (*c*),

$$dU = \frac{1}{2}(\tau_{xy} dxdz)(\gamma_{xy} dy) = \frac{1}{2}\tau_{xy}\gamma_{xy} dV$$

Stress-Strain-Temperature Relations

Criteria for Initial Yielding

Mises yield criterion

According to Mises criterion, yielding occurs in three-dimensional state of stress when the root mean square of the differences between the principal stresses reaches the same value that it has when yielding occurs in a tensile test.

$$\sqrt{\frac{1}{3}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]}=\sqrt{\frac{1}{3}\left[(Y-0)^2+(0-0)^2+(0-Y)^2\right]}=\sqrt{\frac{2}{3}}Y$$

$$\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]}=Y$$

It is also known as distortion- energy criterion or octahedral shear stress criterion for yielding

Stress-Strain-Temperature Relations

Criteria for Initial Yielding

Tresca yield criterion

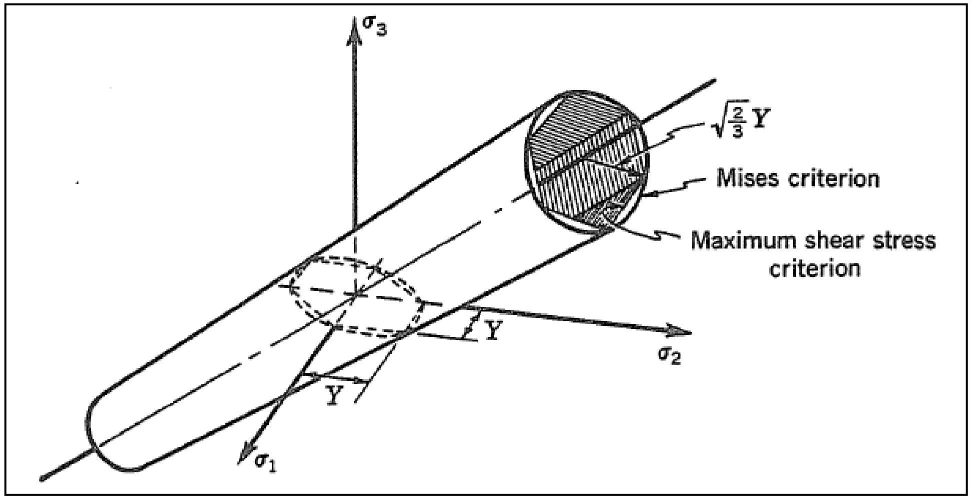
According to Tresca criterion, the yielding occurs when the absolute maximum shear stress at a point reaches the value of the maximum shear stress to cause yielding in a tensile test

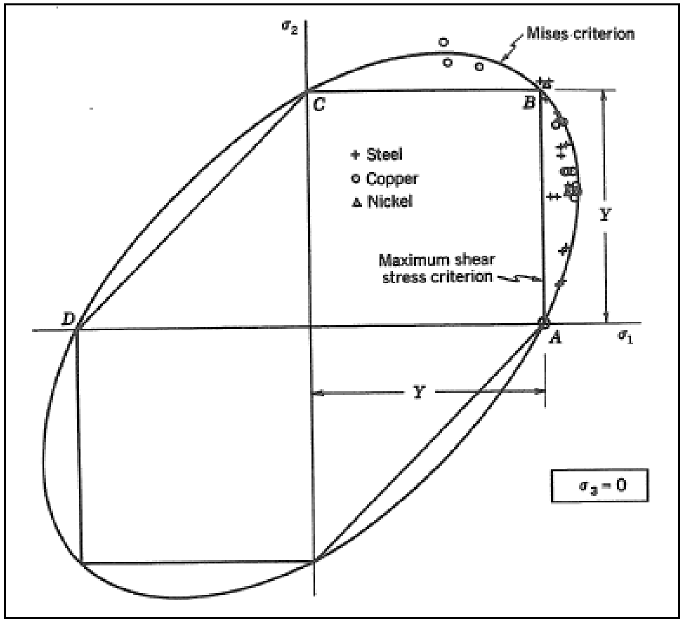
$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{Y}{2}$$

It is also known as maximum shear stress criterion.

Criteria for Initial Yielding

Geometrical representation in principal stress space of the Mises and Tresca yield criteria.





Plane stress case

Stress-Strain-Temperature Relations

Problem

A batch of 2024-T4 aluminum alloy yields in uniaxial tension at stress σ_o = 330MPa. If this material is subjected to the following state of stress, will it yield according to

a. The Mises criterion

b. Tresca criterion

$$\sigma_x = 138\text{MPa} \qquad \tau_{xy} = 138\text{MPa}$$
$$\sigma_y = -69\text{MPa} \qquad \tau_{yz} = 0$$
$$\sigma_z = 0 \qquad \tau_{xz} = 0$$

References

1. Introduction to Mechanics of Solids by S. H. Crandall et al (In SI units), McGraw-Hill